Photogrammetry to Create 3D Models

A Develop Understanding Task

Purpose: In this activity students will develop an understanding of how a photogrammetric engineer uses measurements from a photograph to create a 3-D model of an object on the ground.

Career Field:
Photogrammetry
NCDOT Photogrammetry Unit

WTCC Associate Program of Study and Contact Person:
Civil Engineering and Geomatics Technology
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NC Math 4 Standards:
NC.M4.AF.1.1 Execute algebraic procedures to compose two functions.
NC.M4.AF.1.2 Execute a procedure to determine the value of a composite function at a given value when the functions are in algebraic, graphical, or tabular representations.

Unit Alignment:
NC Math 4 - Unit 2 Functions
WTCC Math 110, 121, 171, or 143

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.

Prerequisite Skills

- Solving literal equations
- Evaluating expressions involving decimals
- Relationship of sides in similar triangles
- Solving problems involving proportions
- Writing basic formulas in Excel

Time Required
The time required to complete this activity is approximately 90 minutes.

Materials Needed

- Excel Spreadsheet
- Student Activity Sheet
- Desmos
- Geogebra for extension (Task 4)

The Teaching Cycle:

Launch:

[Desmos Activity and Launch Video]

In the Desmos Activity students will practice solving literal equations, evaluating expressions involving decimals, relationships in similar triangles and solving proportions. The activity will also go through basic Excel Spreadsheet skills of writing equations. The video introducing Photogrammetry to Create 3-D Models is embedded in the Desmos activity. After the video there are a few questions that review the vocabulary necessary for the activity.

Explore:

Students will begin Photogrammetry to Create 3-D Models Activity by exploring the idea of parallax. Students will explore this idea by extending their thumb (arm's length) and looking at an object across the room with left eye closed and again with the right eye closed. Students will draw what they see and discuss why the images are different. You may want to use a simple picture strategically placed on the wall in your classroom that all groups can see. An optional picture of a drone is included.

Students may be interested in this topic. You may want to bring the class together and have a discussion about eye dominance. Students can discover their eye dominance by doing the following: https://lasikomaha.com/find-dominant-eye

Extend your arms in front of you and create a triangular opening between your thumbs and pointer fingers by placing them together.

1. With both of your eyes open, center this triangle on a distant object, like a clock or picture frame.
2. Close your left eye.
3. If the object stays centered when your left eye is closed, then your right eye (the eye that is open) is your dominant eye. If the object is no longer framed by the triangular opening, your left eye is your dominant eye.

Another extension of this opening activity is to ask students to draw what they see with both eyes opened.

The formula for parallax is $x_1 - x$. These variables represent the x-coordinates of the points that locate the same point in two different photographs. In Figure 2 and Figure 3 x *and* x_1 are labeled in the diagram. Note that it looks like they represent lengths. Reinforce that they represent the x-coordinates of points a' and a'_1 in the photos taken at the two locations L and L_1.

Task 1

In Figure 1 *Point A* is highlighted in both photographs with an x-y axis imposed over them. It is important to note that the x-y axis is fixed and the photograph is what is moving. Think of it as the x-y axis is fixed on the camera. The goal of this activity is to use the coordinates on the photographs to find the **ground coordinates** and the elevation of the object on the ground. The image in the photo is a scaled down version of the object on the ground. We are trying to make the connection to similar triangles and the proportional reasoning.

While students are familiar with graphing in the 2-dimensional x-y plane, they may be unfamiliar with graphing in the x, y, z plane. Using 3 meter sticks or string may provide a concrete representation of 3-dimensional graphing. Geogebra is an online tool that allows students to graph in 3-dimension.

In Task 1 part g) it may be difficult to distinguish the difference between x and X on the diagram. It would be very helpful to project a larger version of the diagram for students to see the labels clearly. There is an enlarged picture provided for your convenience. Figure 2

Figure 1

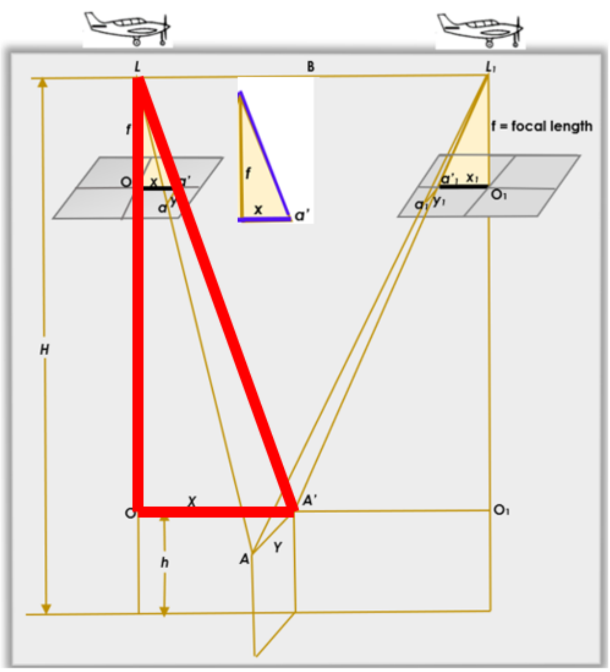
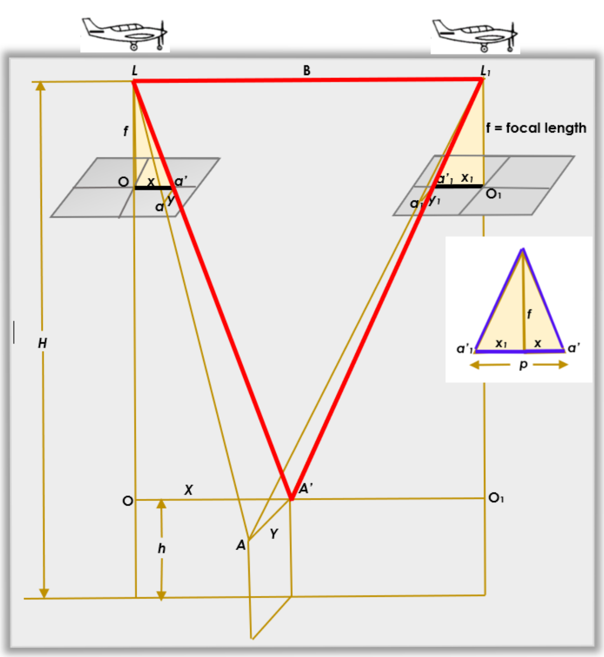


Figure 3 is used for part j). In part j) of Task 1 assume a horizontal flight path & right angles where appropriate. The goal is for students to indicate that the red and blue triangles are similar. This in turn will lead to the proportions in part k). An enlargement of this diagram may be useful for students. Figure 3



In part k) students may need help distinguishing between x and X in Figure 3.

The second proportion may be difficult for students to get on their own. You may need to help students see the similar triangles outlined below. Students may struggle with what to call the vertical red line (will likely call it LO). Since $H - h$ is used in future calculations, you may need to guide students to how the length of LO is calculated if the length of H is known.

In Task 2 students are walked through how to find the elevation of a point and how to find the ground coordinates of a point. The equations from Task 1 part k) are used to do this. While students have not been specifically given the formula for finding the Y value of the ground coordinate, it can be found by substituting x and X in the equation with y and Y. Once students have found this, reinforce the idea that these are 3-D coordinates (x, y, z).

For Task 3 students will need to put simple equations into an Excel spreadsheet. The Desmos activity covers the basic functions that they will use.
Students may need support on how to write the equations due to double parantheses necessary

parallax formula =(C24-F24)
h =4000-(6*900)/G24
X =((C24*(4000-K24))/6)
Y =((D24*(4000-K24))/6)

Task 4 has students graph the coordinates that they found in Task 3 to create a 3-D model of the road and embankment. It is advantageous to divide the coordinates by 100 to scale the model down so that it will fit on the axes. Geoalgebra or Excel can be used to generate the 3-D graphic from the (x,y,z) coordinates determined in the spreadsheet.

Discuss:

Exit Ticket:

Ask students how might the recreation of the 3-dimensional model be used in the real world?

An example assessment for testing:

Photos are taken with an air base 1000 feet apart from each other with a camera that has a focal length of 5.5 inches by an aircraft flying 3000 feet above sea level. A certain object on the ground can be seen from overlapping photographs and the image space coordinates of that object in photo 1 is $(27.649\ in, 19.825\ in)$ and the x-coordinate of the object in photo 2 is $29.794\ in$.

a) Draw a picture that illustrates each value provided in this description.
b) Find the space coordinates of the object.